Rotational Motion

Question1

Due to global warming, if the ice in the polar region melts and some of this water flows to the equatorial region, then

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Options:

A.

angular momentum of the Earth increases and duration of day increases.

B.

angular momentum of the Earth decreases and duration of day decreases.

C.

angular momentum of the Earth is constant and duration of day decreases.

D.

angular momentum of the Earth is constant and duration of day increases.

Answer: D

Solution:

When ice from the polar regions melts and flows towards the equator, the mass distribution of the Earth changes. This shift in mass causes in increases in the Earth's moment of inertia. According to the principle of conservation of angular momentum, if the angular momentum ($L = I\omega$) remains constant and the moment of inertia (I) increases, the angular velocity (ω) must decrease. A decrease in angular velocity means the Earth rotates more slowly, leading to an increase in the duration of a day.

Question2

If the moment of inertia of a thin circular ring about an axis passing through its edge and perpendicular to its plane is I, then the moment of inertia of the ring about its diameter is

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Options:

A.

$I/4$

B.

$4I$

C.

$I/2$

D.

$2I$

Answer: A

Solution:

Moment of inertia of ring about its center:

$I_{\text{CM}} = MR^2$

Using the parallel axis theorem:

The parallel axis theorem says:If you know the moment of inertia of an object about its center (I_{CM}), then the moment of inertia about a parallel axis a distance R away is:

$I = I_{\text{CM}} + MR^2$

For a thin circular ring, this axis through the edge is a distance R from the center, so:

$I = MR^2 + MR^2 = 2MR^2$

So, $MR^2 = \frac{I}{2} \qquad \cdots (i)$

Finding the moment of inertia about the diameter:

The moment of inertia of a ring about its diameter is:

$I_d = \frac{MR^2}{2}$

Substituting from equation (i):

Replace MR^2 with $\frac{I}{2}$:$I_d = \frac{\frac{I}{2}}{2} = \frac{I}{4}$

Question3

If the moment of inertia of a uniform solid cylinder about the axis of the cylinder is $\frac{1}{n}$ times its moment of inertia about an axis passing through its midpoint and perpendicular to its length, then the ratio of the length and radius of the cylinder is

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Options:

A.

$\sqrt{2(3n+1)}$

B.

$\sqrt{2(3n-1)}$

C.

$\sqrt{3(2n+1)}$

D.

$\sqrt{3(2n-1)}$

Answer: D

Solution:

As we know,

Moment of inertia about axis of cylinder,

$I_{\text{long}} \quad = \frac{1}{2} MR^2$

Moment of inertia about an axis through centre and perpendicular to length.

$I_{\text{transverse}} \quad = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$

By using, given ratio,

$$\frac{1}{2} M R^2=\frac{1}{n}\left(\frac{1}{12} M L^2+\frac{1}{4} M R^2\right)$$

$$\left(\frac{2 n-1}{4}\right) R^2=\frac{1}{12} L^2$$

$$3(2 n-1)=\frac{L^2}{R^2} \Rightarrow \frac{L}{R}=\sqrt{3(2 n-1)}$$

Question4

A body of mass m and radius r rolling horizontally m an inclined plane to a vertical

a velocity v rolls up an height $\frac{v^2}{g}$. The body is

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Options:

A. a sphere

B. a circular disc

C. a circular ring

D. a solid cylinder

Answer: C

Solution:

Let moment of inertia of body $=I$

The initial kinetic energy $=\frac{1}{2} I \omega^2+\frac{1}{2} m v^2$

Final potential energy $=m g h$

Given, $h=\frac{v^2}{g}$

After putting value, potential energy

$=m \cdot g \cdot \frac{v^2}{g}=m v^2$

When a body roll up an inclined plane of height h, its kinetic energy converted into potential energy.

So, KE = PE

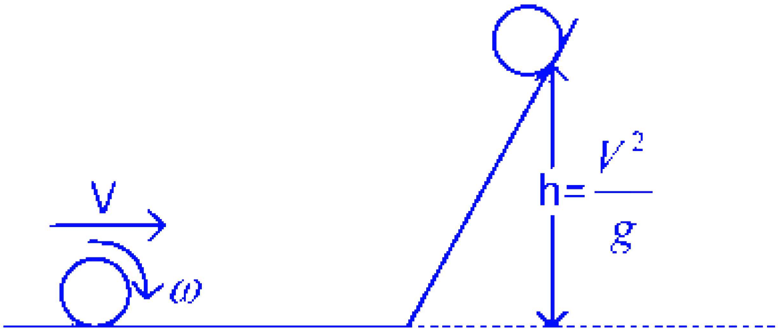
$\frac{1}{2} I \omega^2+\frac{1}{2} m v^2=m v^2$

$\frac{1}{2} I \omega^2=\frac{1}{2} m v^2$

$I=\frac{m v^2}{\omega^2}=\frac{m(r \omega)^2}{\omega^2}$

$I=m r^2$

So, body is a ring.



Question5

A hollow cylinder and a solid cylinder initially at restat the top of an inclined plane are rolling down without slipping. If the time taken by the hollow cylinder to reach the bottom of the inclined plane is 2 s , the time taken by the solid cylinder to reach the bottom of the inclined plane is

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Options:

A. 2 s

B. 1.414 s

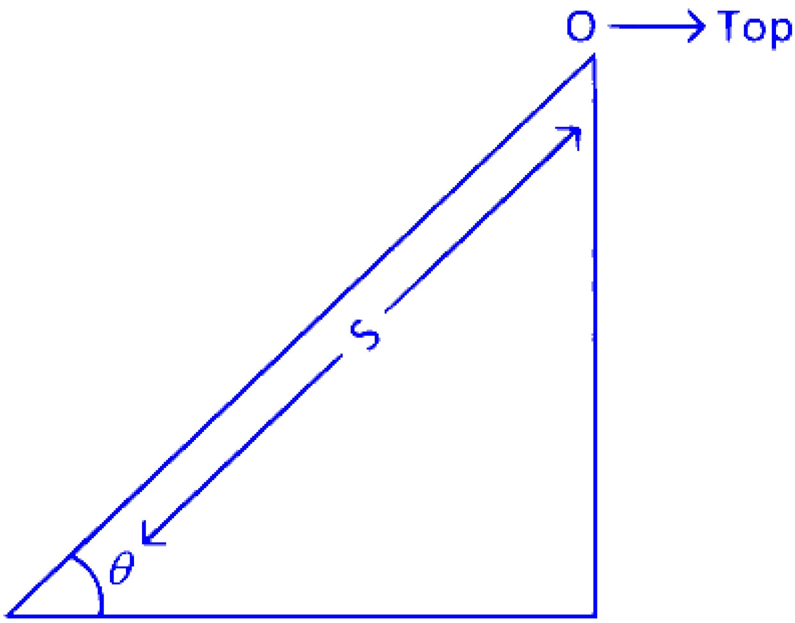
C. 1 s

D. 1.732 s

Answer: D

Solution:

Time taken by body to roll down from a slope without slipping is given by,



$t=\sqrt{\frac{2s}{g\sin\theta\beta}}$

where $\beta=\frac{1}{1+\frac{K^2}{R^2}}$

From Eq. (i)

$t\propto\frac{1}{\beta}$

$t_1=$ time to reach bottom (by hollow cylinder)

$t_2=$ time to reach bottom (by solid cylinder)

$\frac{t_1}{t_2}=\sqrt{\frac{\beta_2}{\beta_1}}$

β_1, for hollow cylinder

$\beta_1=\frac{1}{1+\frac{K^2}{R^2}}=\frac{1}{1+1}=\frac{1}{2}\quad\left[\text{as } \frac{K^2}{R^2}=1\right]$

For solid cylinder

$\beta_2=\frac{1}{1+\frac{1}{2}}=\frac{2}{3}$

Substitute all value in Eq. (iv)

$$\frac{2}{t_2}=\sqrt{\frac{\frac{2}{3}}{\frac{1}{2}}} \Rightarrow \frac{2}{t_2}=\sqrt{\frac{4}{3}}$$

$$t_2=2 \times \sqrt{\frac{3}{4}}$$

$$t_2=1.732 \mathrm{~s}$$

Question6

A solid sphere and a disc of same mass M and radius R - are kept such that their curved surfaces are in contact and their centres lie along the same horizontal line. The moment of inertia of the two body system about an axis passing through their point of contact and perpendicular to the plane of the disc is

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Options:

A. $\frac{53 M R^2}{20}$

B. $\frac{39 M R^2}{10}$

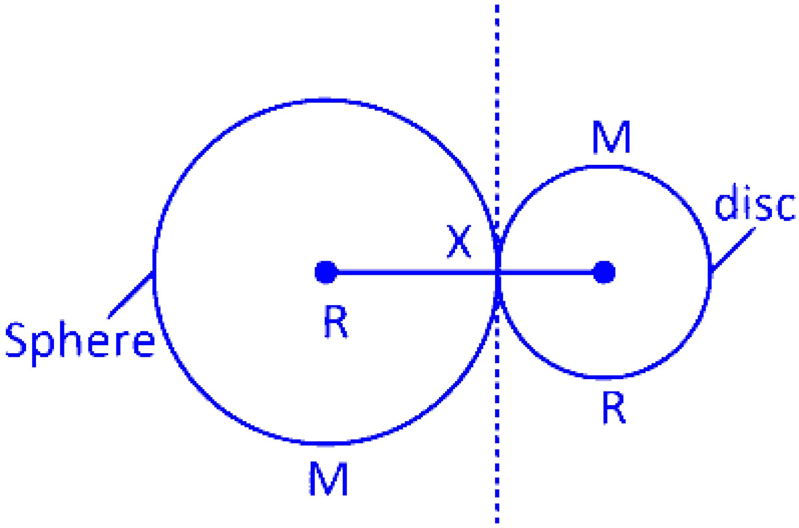
C. $\frac{29 M R^2}{10}$

D. $\frac{9 M R^2}{10}$

Answer: C

Solution:

Disc and sphere has same radius and mass.



Now, moment of inertia of sphere at centre (I)

$I = \frac{2MR^2}{5}$

Using parallel axis theorem, moment of inertia at point X of sphere

$I_{XS} = I + MR^2$

$$= \frac{2}{5}MR^2 + MR^2 = \frac{7MR^2}{5}$$

Moment of inertia of disc at centre perpendicularly

$I = \frac{MR^2}{2}$

Using parallel axis theorem, moment of inertia at point X of disc.

$I_{\chi D} = I + MR^2 = \frac{3MR^2}{2}$

Total moment of inertia at point X

$$I_{XS} + I_{XD} = \frac{3MR^2}{2} + \frac{7MR^2}{5}$$

$$= \frac{29MR^2}{10}$$

Question7

A thin uniform wire of mass m and linear mass density ρ is bent in the form of a circular loop. The moment of inertia of the loop about its diameter is

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Options:

A. $\frac{m^2}{4\pi^2\rho^2}$

B. $\frac{m^2}{4\rho^2}$

C. $\frac{m^3}{8\pi^2\rho^2}$

D. $\frac{m^2}{8\rho^2}$

Answer: A

Solution:

Given, Mass of wire, $= m$

linear mass density $= \rho$

Let length of wire be l and radius ber

$\rho = \frac{m}{l}$

or

$m = \rho l \quad \ldots \text{(i)}$

also,

$2\pi r = 1$

$$\Rightarrow r = \frac{1}{2\pi} \quad \ldots \text{(ii)}$$

$\therefore$ The moment of inertia around a circular loop around COM, will be

$$I = \frac{1}{2}mr^2$$

$$\left. I = \frac{1}{2}(\rho l)\left(\frac{l}{2\pi}\right)^2 \{\text{using Eqs. (i) and (ii)}\} \right\}$$

$$= \frac{1}{2}\rho\frac{l^3}{4\pi^2}$$

$$= \frac{1}{8\pi^2}\rho l^3$$

From parallel axis theorem,

$$I_{\text{diameter}} = I_{\text{COM}} + MR^2$$

$$= \frac{l^3\rho}{8\pi^2} + \frac{l^3\rho}{4\pi^2}$$

$$= \frac{3l^3\rho}{8\pi^2}$$

or $$I_{\text{diameter}} = \frac{3}{8}\frac{m^3}{\pi^2\rho^2}$$

Question8

A solid sphere rolls down without slipping from the top of an inclined plane of height 28 m and angle of inclination 30°. The velocity of the sphere, when it reaches the bottom of the plane is (Acceleration due to gravity $= 10 \text{ ms}^{-2}$)

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Options:

A. 20 ms^{-1}

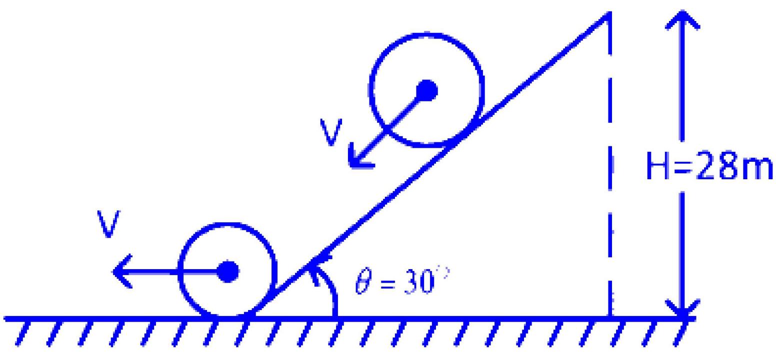
B. 10 ms^{-1}

C. 28 ms^{-1}

D. 14 ms^{-1}

Answer: A

Solution:



Given, height of plane $= 28$ m

angle of inclination, $\theta = 30^{\circ}$

By conservation of energy,

Potential energy lost by the solid sphere in rolling down the inclined plane

$=$ kinetic energy gain by the sphere

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$mgh = \frac{7}{10}mv^2$$

$$v^2 = \frac{10}{7}gh = \sqrt{\frac{10}{7} \times 10 \times 28}$$

$$v = 20 \text{ m/s}$$

The velocity of the sphere, when it reaches the bottom of the plane is 20 m/s.

Question9

Moon revolves around the earth in an orbit of radius R with time period of revolution T. It also rotates about its own axis with a time period T. If mass of the moon is M and its radius is r, the total kinetic energy of the moon is

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Options:

A. $\frac{2M\pi^2R^2}{T^2} + \frac{4Mr^2\pi^2}{5T^2}$

B. $\frac{M\pi^2R^2}{2T^2}$

C. $\frac{4Mr^2\pi^2}{5T^2}$

D. $\frac{M\pi^2R^2}{2T^2} + \frac{4Mr^2\pi^2}{5T^2}$

Answer: A

Solution:

To determine the total kinetic energy of the moon, we consider both its revolution around the Earth and its rotation about its own axis. Here are the given parameters:

The moon revolves around the Earth in an orbit with radius R.

The time period of this revolution is T.

The radius of the moon is r.

The mass of the moon is M.

The total kinetic energy can be divided into two components:

Kinetic energy due to revolution around the Earth:

The formula for rotational kinetic energy is $\frac{1}{2}I\omega^2$.

For revolution around the Earth, the moment of inertia is $I = MR^2$.

The angular velocity ω is $\frac{2\pi}{T}$.

Substituting these values, we find:

Kinetic energy (revolution) $= \frac{1}{2}(MR^2)\left(\frac{2\pi}{T}\right)^2 = \frac{2M\pi^2R^2}{T^2}$

Kinetic energy due to rotation about its own axis:

For rotation on its own axis, the moment of inertia is $I = \frac{2}{5}Mr^2$.

Again, the angular velocity ω is $\frac{2\pi}{T}$.

Substituting these values, we find:

Kinetic energy (rotation) $= \frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\frac{2\pi}{T}\right)^2 = \frac{4Mr^2\pi^2}{5T^2}$

Finally, to find the total kinetic energy of the moon, we sum both components:

Total kinetic energy $= \frac{2M\pi^2R^2}{T^2} + \frac{4Mr^2\pi^2}{5T^2}$

Question10

The spinning of the Diwali cracker 'ground chakkar' involves the concept of

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Options:

A. conservation of mechanical energy

B. conservation of linear momentum

C. conservation of angular momentum

D. conservation of charge

Answer: C

Solution:

The spinning of a Diwali cracker, known as a 'ground chakkar,' demonstrates the principle of conservation of angular momentum.

The formula for angular momentum is given by:

$I\omega = \text{constant}$

which can be expressed as:

$I_{\text{Initial}} \quad \omega_{\text{Initial}} \quad = I_{\text{Final}} \quad \omega_{\text{Final}}$

Where:

I is the moment of inertia and ω is the angular velocity.

Initially, let's denote:

Radius of the chakkar as R.

Mass of the chakkar as M.

Assume the surface on which the chakkar rotates is smooth.

After some time:

Final radius becomes R'.

Final mass is M'.

Initial angular speed is ω_i.

Final angular speed is ω_f.

The formula for the moment of inertia for the chakkar is:

$I = \frac{mR^2}{2}$

Substituting this into the conservation equation:

$M_i\left(\frac{R_i^2}{2}\right)\omega_i = M_f\left(\frac{R_f^2}{2}\right)\omega_f$

Given that:

$M_i > M_f$

$R_i > R_f$

It follows that:

$\omega_f > \omega_i$

This explains why the chakkar begins with a slower angular speed and then accelerates as it burns, losing mass and thus increasing its angular speed.

Question11

If the radius of the earth becomes x times its present value, the new period of rotation in hours is

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Options:

A. $6x^2$

B. $12x^2$

C. $24x^2$

D. $48x^2$

Answer: C

Solution:

To determine how the period of the Earth's rotation would change if its radius became x times its current value, consider the following:

Let the original radius of Earth be R and its mass be M.

Increase the radius: If the radius is increased to xR, we need to apply the conservation of angular momentum because there are no external torques acting on the Earth:

$$I_1\omega_1 = I_2\omega_2$$

Moment of Inertia: For a sphere, the moment of inertia I is given by:

$$I = \frac{2}{5}MR^2$$

where M is the Earth's mass and R is its radius. The angular frequency ω is related to the period T by $\omega = \frac{2\pi}{T}$.

Substitute into the Conservation Equation: Initially, the moment of inertia and angular frequency are:

$$I_1 = \frac{2}{5}MR^2, \quad \omega_1 = \frac{2\pi}{24 \text{ hours}}$$

New Moment of Inertia and Angular Frequency: With the expanded radius:

$$I_2 = \frac{2}{5}M(xR)^2 = \frac{2}{5}Mx^2R^2$$

New Period Calculation: Substitute these into the conservation equation:

$$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{24 \text{ hours}} = \left(\frac{2}{5}Mx^2R^2\right)\frac{2\pi}{T}$$

Simplifying this equation for T gives:

$$T = 24x^2$$

Thus, if the radius of the Earth becomes x times its original value, the new period of rotation will be $24x^2$ hours.
